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Instability of the ionization-absorption balance in a complex plasma at ion time scales

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The stability of ion plasma perturbations is investigated in a homogeneous isotropic complex plasma, where a balance between plasma creation due to ionization and plasma loss due to the absorption on dust particles has been reached. The analysis is performed on the basis of a self-consistent fluid description including dust charge variations and ion-neutral friction. It is shown that the stability depends primarily on the nature of the ionization source. For an ionization source proportional to the electron density, an instability takes place at wave numbers below a certain threshold, and the instability mechanism is explained in detail. No instability is found for a constant ionization source.

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I. INTRODUCTION

A complex plasma is a plasma containing dust particles [1,2]. The dust grains in a plasma environment become electrically charged because of the absorption of ions and electrons. For typical laboratory and industrial plasmas, the net charge of a dust grain is negative because of higher mobility of plasma electrons [3]. The variability of the dust charge with electron and ion densities as well as the absorption of ions and electrons by the dust leads to effects not seen in ordinary electron-ion plasmas [3,4].

An important example of the specific complex plasma phenomena is the collective attraction, when negatively charged dust particles attract each other due to the overscreening of their electrostatic potentials by plasma ions and electrons [4]. The collective attraction was studied in Refs. [5–9]; the general approach adopted was to consider the screening of the Coulomb potential of a charged dust particle by ions and electrons in the presence of the plasma absorption on other dust grains and ionization compensating the absorption globally [4]. The obtained resulting potential has, apart from the Debye core, a longer ranging attractive component.

However, the derivations of Refs. [5–9] are based on an implicit assumption that such an ionization-absorption-balanced plasma is stable with respect to perturbations that can appear at ion time scales. That is, the ion component was assumed to be in a stable equilibrium state and then the screening of a test dust particle was considered as a static perturbation of this state.

In the present paper, we address the validity of this assumption by studying the stability of a homogeneous ionization-absorption-balanced complex plasma at ion time scales. Because the collective attraction was derived both for a constant ionization source [8,9] and an ionization source proportional to the electron density [5–7], we employ a general model with an arbitrary dependence of the ionization source on the electron density. This, as well as the self-consistent account for dust charge variations, distinguishes our investigation from other studies of ion plasma waves in complex plasmas [10–13]. We show that the plasma is unstable under certain conditions and explain in detail the instability mechanism, as well as compare the instability length with the characteristic scale of the collective attraction. Based on these results, we assess the validity of the previous investigations of the collective attraction.

II. MODEL

A. Fluid equations

We consider a weakly ionized complex plasma consisting of neutral particles, positive ions, electrons, and negatively charged dust grains. The dust grains are considered as a continuous, immovable, and uniform "dust medium" with constant number density n_{d}, whereas the ions and electrons have number densities n_{i} and n_{e}, respectively, which may vary with time and space. The dust charge $-eZ_{\mathrm{d}}$ is governed by the absorption of ions and electrons and hence may vary with space and time as well. Here, $e>0$ is the elementary charge.

We use the orbit motion limited (OML) expression for the electron flux onto a single dust particle [1–4], whereas for ions we assume an arbitrary dependence of the absorption rate on the dust charge since it has been demonstrated that the OML model does not work well for ions in many laboratory complex plasmas [14]. Hence, the dust charging equation becomes

$$n_{\mathrm{d}}\frac{\partial Z_{\mathrm{d}}}{\partial t} = -\alpha_{\mathrm{a}}(Z_{\mathrm{d}})n_{\mathrm{i}} + \sqrt{8\pi}a^2 v_{\mathrm{te}} n_{\mathrm{e}} n_{\mathrm{d}} \exp\left(-\frac{Z_{\mathrm{d}}e^2}{aT_{\mathrm{e}}}\right), \quad (1)$$

where a is the grain radius, T_{e} is the (constant) electron temperature, $v_{\mathrm{te}}=\sqrt{T_{\mathrm{e}}/m_{\mathrm{e}}}$ is the electron thermal velocity, m_{e} is the electron mass, and $\alpha_{\mathrm{a}}(Z_{\mathrm{d}})$ is the ion absorption rate. We note that, in general, the ion absorption rate depends on the ion fluid velocity $\mathbf{v}$ as well; but this dependence cannot contribute to our results because we will consider here only first-order perturbations of a state with $\mathbf{v}=\mathbf{0}$.

The ions are produced by plasma ionization at a rate per unit volume $S(n_{\mathrm{e}})$ which we take to depend generally on n_{e}. The ions are absorbed by the dust at a rate per unit volume $\alpha_{\mathrm{a}}(Z_{\mathrm{d}})n_{\mathrm{i}}$. We neglect other ion sinks, such as recombination, and hence the ion continuity equation takes the form

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \nabla\cdot(n_{\mathrm{i}}\mathbf{v}) = -\alpha_{\mathrm{a}}(Z_{\mathrm{d}})n_{\mathrm{i}} + S(n_{\mathrm{e}}). \quad (2)$$

The ions experience an electric field force and a friction force arising from collisions with neutrals, Coulomb and charging collisions with dust particles, and ionization. Hence, the ion momentum equation is

$$m_{\mathrm{i}}\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v}\right]=-e\,\boldsymbol{\nabla}\,\varphi-m_{\mathrm{i}}\nu\mathbf{v}-\frac{T_{\mathrm{i}}}{n_{\mathrm{i}}}\boldsymbol{\nabla}\,n_{\mathrm{i}},$$

$$\nu=\nu_{\mathrm{in}}+\nu_{\mathrm{id}}+S(n_{\mathrm{e}})/n_{\mathrm{i}}, \tag{3}$$

where T_{i} is the (constant) ion temperature, m_{i} is the ion mass, φ is the electrostatic potential, and ν is the friction coefficient which is the sum of the effective ion-neutral collision frequency ν_{in}, effective ion-dust collision frequency ν_{id}, and the ionization term $S(n_{\mathrm{e}})/n_{\mathrm{i}}$ [15]. The dependence of the friction coefficient ν on plasma parameters is not important to our results because we will consider here only first-order perturbations of a state with $\mathbf{v}=\mathbf{0}$.

The electron inertia can be neglected on the ion time scales, thus, leading to the Boltzmann distribution for plasma electrons,

$$e\,\boldsymbol{\nabla}\,\varphi-\frac{T_{\mathrm{e}}}{n_{\mathrm{e}}}\boldsymbol{\nabla}\,n_{\mathrm{e}}=\mathbf{0}. \tag{4}$$

Finally, the electric field is related to the charge density by the Poisson's equation

$$-\nabla^2\varphi=4\pi e(n_{\mathrm{i}}-n_{\mathrm{e}}-n_{\mathrm{d}}Z_{\mathrm{d}}). \tag{5}$$

B. Equilibrium

We assume the existence of a uniform equilibrium solution of Eqs. (1)–(5),

$$n_{\mathrm{i}},n_{\mathrm{e}},Z_{\mathrm{d}}=\mathrm{const},\quad \mathbf{v}=\mathbf{0},\quad \varphi=0. \tag{6}$$

The equilibrium is characterized by the following dimensionless parameters:

(i) The Havnes parameter $P=n_{\mathrm{d}}Z_{\mathrm{d}}/n_{\mathrm{i}}$.
(ii) The friction parameter $\zeta=\nu/\omega_{\mathrm{pi}}$.
(iii) The absorption parameter $\eta=\alpha_a(Z_{\mathrm{d}})/\omega_{\mathrm{pi}}$.
(iv) The normalized dust charge $z=Z_{\mathrm{d}}e^2/(aT_{\mathrm{e}})$.
(v) The temperature ratio $\tau=T_{\mathrm{i}}/T_{\mathrm{e}}$.
(vi) The parameter

$$\beta=\frac{d\alpha_{\mathrm{a}}(Z_{\mathrm{d}})}{dZ_{\mathrm{d}}}\frac{Z_{\mathrm{d}}}{\alpha_a(Z_{\mathrm{d}})}, \tag{7}$$

characterizing the dependence of the ion absorption rate on the dust charge.

(vii) The parameter

$$\gamma=\frac{dS(n_{\mathrm{e}})}{dn_{\mathrm{e}}}\frac{n_{\mathrm{e}}}{S(n_{\mathrm{e}})} \tag{8}$$

characterizing the nature of the ionization source ($\gamma=1$ for an ionization source proportional to the electron density, whereas $\gamma=0$ for a constant ionization source).

Here, $\omega_{\mathrm{pi}}=\sqrt{4\pi n_{\mathrm{i}}e^2/m_{\mathrm{i}}}$ is the ion plasma frequency.

C. Stability analysis

To analyze the stability of the equilibrium, we consider first-order perturbations in the form $\propto\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$ with the real wave vector $\mathbf{k}$ and the complex frequency ω. We introduce the normalized frequency $\tilde{\omega}=\omega/\omega_{\mathrm{pi}}$ and the normalized wave number $\tilde{k}=k\lambda_{\mathrm{Di}}$, where $k=|\mathbf{k}|$, $\lambda_{\mathrm{Di}}=v_{\mathrm{ti}}/\omega_{\mathrm{pi}}$ is the ion Debye length, and $v_{\mathrm{ti}}=\sqrt{T_{\mathrm{i}}/m_{\mathrm{i}}}$ is the ion thermal velocity.

III. RESULTS

A. Linearized equations

The set of linearized equations for the first-order perturbations takes the form

$$\mathcal{M}\mathcal{L}=0, \tag{9}$$

where

$$\mathcal{L}=\left(\frac{\delta n_{\mathrm{i}}}{n_{\mathrm{i}}},\frac{\delta n_{\mathrm{e}}}{n_{\mathrm{i}}},\frac{e^2\delta Z_{\mathrm{d}}}{aT_{\mathrm{e}}},\frac{\delta v_x}{v_{\mathrm{ti}}},\frac{e\,\delta\varphi}{T_{\mathrm{i}}}\right)^T, \tag{10}$$

δn_{i}, δn_{e}, etc., denote the corresponding perturbation amplitudes, δv_x is the perturbation amplitude of the velocity component along the wave vector $\mathbf{k}=(k,0,0)$, and

$$\mathcal{M}=\begin{pmatrix} \eta-i\tilde{\omega} & -\dfrac{\eta\gamma}{1-P} & \dfrac{\eta\beta}{z} & i\tilde{k} & 0 \\ i\tilde{k} & 0 & 0 & \zeta-i\tilde{\omega} & i\tilde{k} \\ 0 & -1 & 0 & 0 & \tau(1-P) \\ 1 & -\dfrac{1}{1-P} & 1+\dfrac{\beta}{z}-\dfrac{i\tilde{\omega}P}{\eta z} & 0 & 0 \\ -1 & 1 & \dfrac{P}{z} & 0 & \tilde{k}^2 \end{pmatrix}. \tag{11}$$

Note that equations for perturbations of the perpendicular velocity components $\delta v_{y,z}$ are decoupled from Eq. (9) and take the form $(\zeta-i\tilde{\omega})\delta v_{y,z}=0$, which gives the trivial mode $\tilde{\omega}=-i\zeta$.

B. Dispersion relation

The dispersion relation $\det\mathcal{M}=0$ takes the form

$$i\tilde{\omega}^3(c_1\tilde{k}^2+c_2)+\tilde{\omega}^2(c_3\tilde{k}^2+c_4)+i\tilde{\omega}(c_5\tilde{k}^4+c_6\tilde{k}^2+c_7)+c_8\tilde{k}^4+c_9\tilde{k}^2+c_{10}=0, \tag{12}$$

where the coefficients c_j are

$$c_1=-\frac{P}{\eta z},$$

$$c_2=-\frac{P\tau(1-P)}{\eta z},$$

$$c_3=1+\frac{P+\beta}{z}+\frac{P\zeta}{\eta z},$$

$$c_4 = \tau\left[1 + \frac{\beta}{z} + \frac{P}{z}\left(1 + \frac{\zeta}{\eta}\right)\right](1-P) + \frac{\tau P}{z}(1-\gamma),$$

$$c_5 = P/(\eta z),$$

$$c_6 = \zeta\left(1 + \frac{\beta+P}{z}\right) + \eta + \frac{P}{\eta z}[1 + \tau(1-P)],$$

$$c_7 = \tau\eta\left[\frac{P\beta}{z} - \gamma\left(1 + \frac{P+\beta}{z}\right)\right] + \tau(\zeta+\eta) \times\left[\left(1 + \frac{\beta}{z}\right)(1-P) + \frac{P}{z}\right] + \frac{P\tau\zeta}{z}(1-P-\gamma),$$

$$c_8 = -\left(1 + \frac{\beta}{z}\right),$$

$$c_9 = -\left(1 + \frac{\beta}{z}\right)[\tau(1-P)+1] - (1+\tau)\frac{P}{z} - \zeta\eta,$$

$$c_{10} = \tau\eta\zeta\left[\left(1 + \frac{P+\beta}{z}\right)(\gamma-1) + P\right]. \tag{13}$$

Equation (12) is cubic with respect to $\tilde{\omega}$ due to the time-derivative terms in the ion momentum, ion continuity, and dust charging equations. Thus, two of the three modes originate from the ion dynamics explicitly, and the remaining mode originates from the dynamics of dust charging.

C. Limit $k \to 0$

Let us consider the behavior of the dispersion relation at $k \to 0$. In this limit, the solutions of Eq. (12) are $\tilde{\omega}=-i\zeta$ and

$$\tilde{\omega} = -\frac{i\eta}{2}\left(1 + \frac{\beta+z}{P} + \frac{1-\gamma}{1-P}\right) \pm \frac{i\eta}{2}\sqrt{\left(1 + \frac{\beta+z}{P} - \frac{1-\gamma}{1-P}\right)^2 + \frac{4z}{1-P}}. \tag{14}$$

The instability condition at $k \to 0$ then becomes

$$\left(1 + \frac{P+\beta}{z}\right)(\gamma-1) + P > 0. \tag{15}$$

Whether this condition is met depends heavily on the nature of the ionization source. In the case $\gamma=1$, which corresponds to an ionization source proportional to the electron density, inequality (15) is always true (except for the degenerate case of $P=0$); whereas in the case $\gamma=0$, which corresponds to a constant ionization source, inequality (15) is never satisfied because in this paper we assume a negative dust charge (i.e., positive P and z) and positive β. Where γ is between 0 and 1, which can correspond to a combination of different plasma ionization sources, condition (15) may or may not be met (Fig. 1 demonstrates this for typical parameters of laboratory experiments [1–4]).

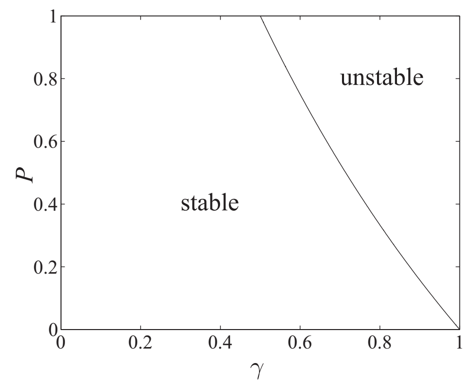

FIG. 1. The stable and unstable regions at small wave numbers ($k \to 0$) and $\beta=1$ and $z=2$ as functions of the Havnes parameter P and the parameter γ characterizing the nature of the ionization source.

D. Limit $k \to \infty$

Let us now consider the behavior of dispersion relation (12) for $k \to \infty$. We obtain a pure imaginary solution

$$\tilde{\omega} = -\frac{i\eta}{P}(\beta+z), \tag{16}$$

and two complex solutions, with the real parts of the opposite sign and the imaginary part

$$\mathrm{Im}(\tilde{\omega}) = -\frac{1}{2}(\eta+\zeta). \tag{17}$$

We see that these solutions are stable.

E. Finite k

The numerically calculated imaginary parts of the mode frequencies for typical experimental parameters [1–4] are shown in Fig. 2. We see that for an ionization source proportional to the electron density, the imaginary part of the unstable mode crosses zero at a certain threshold wave number. For a constant ionization source, all modes remain stable at all wave numbers.

It can be seen from Fig. 2 that at the threshold wave number, all three modes have different imaginary parts and hence have zero real parts [as Eq. (12) is a polynomial in $i\omega$ with real coefficients, it always has either three purely imaginary roots or one purely imaginary root and two complex roots with identical imaginary parts and real parts which are opposite in sign]. Therefore, the threshold wave number can be found analytically by setting $\tilde{\omega}=0$ in dispersion relation (12). By doing this and assuming $\zeta\eta \ll 1$, $\tau \ll 1$, $\beta \sim 1$, $z \sim 1$ (this is well satisfied in laboratory experiments [1–4]), we derive the threshold wave number,

$$\tilde{k} \approx \sqrt{\tau\eta\zeta\left(\frac{Pz}{P+\beta+z} + \gamma - 1\right)}. \tag{18}$$

The expression under the square root here is positive only when condition (15) is met.

Figure 3 shows the unstable mode for $\gamma=1$ and different values of τ. It can be seen that the threshold wave number is

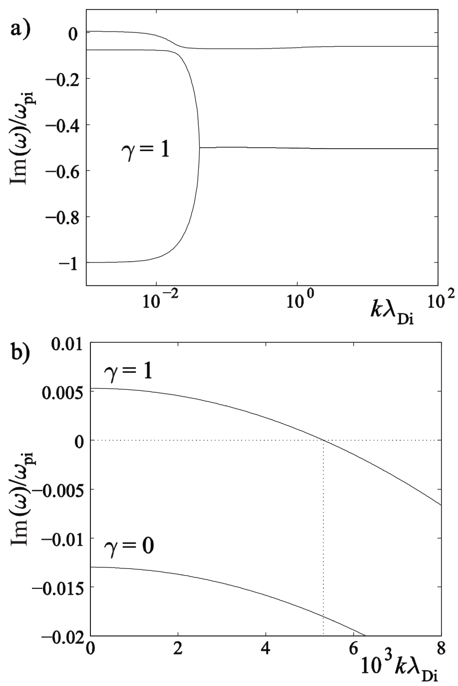

FIG. 2. The imaginary parts of the mode frequencies as functions of the wave number for $P=0.5$, $\zeta=1$, $\eta=0.01$, $\tau=0.01$, $\beta=1$, and $z=2$. (a) The graph shows the case of $\gamma=1$ (ionization source proportional to electron density). For small wave numbers, one of the modes is unstable, as can be seen more clearly in (b) the graph. For $\gamma=0$ (constant ionization source), this instability disappears. For wave numbers larger than those shown in (b) the graph, there is no notable difference in the behavior for $\gamma=0$ and $\gamma=1$, so that all modes for $\gamma=0$ are stable.

$\propto\sqrt{\tau}$, which is in accordance with Eq. (18). Also, the instability growth rate at $k\to 0$ is independent of τ, which is in accordance with Eq. (14).

IV. DISCUSSION

A. Instability mechanism

To understand the physics of the instability, we consider the simplest limiting case where the instability still takes place: $\tilde{k}\to 0$ and $z\to\infty$ (at a fixed P). In this case, the unstable solution of Eq. (12) is

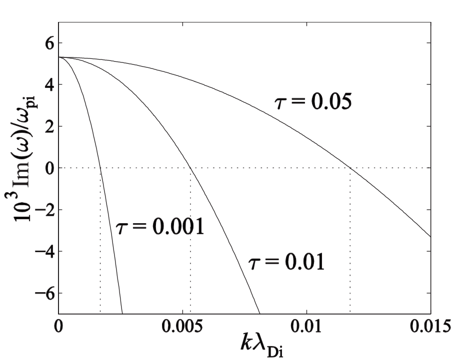

FIG. 3. The unstable mode for three different values of τ, with $\gamma=1$, $P=0.5$, $\zeta=1$, $\eta=0.01$, $\beta=1$, and $z=2$.

$$\tilde{\omega}=i\eta\frac{\gamma-1+P}{1-P} \tag{19}$$

(see also, e.g., Refs. [12,13]). Physically, this solution originates from the ion continuity and Poisson's equations which in this limit (in dimensional units) are, respectively,

$$-i\omega\delta n_{\mathrm{i}}=-\alpha_{\mathrm{a}}\delta n_{\mathrm{i}}+\frac{dS}{dn_{\mathrm{e}}}\delta n_{\mathrm{e}},\quad \delta n_{\mathrm{i}}-\delta n_{\mathrm{e}}=0. \tag{20}$$

Hence, the instability still takes place in the absence of dust charge variations, and the instability mechanism is as follows. A small initial increase in the ion density leads to an equal increase in the electron density because of the quasineutrality, which, in turn, leads to an increase in the ion production due to plasma ionization. If the ionization source is proportional to the electron density, the ion production is increased more than the ion absorption (which is proportional to the ion density) because the *relative* increase in the electron density is larger since $n_{\mathrm{i}}>n_{\mathrm{e}}$ in the equilibrium. This triggers the instability.

B. Instability length

Let us consider the suppression of the instability at threshold wave number (18). We assume for simplicity $\gamma=1$, $P\sim 1-P\sim 1$ (i.e., P is not very close to 0 or 1), and we neglect dust charge variations (the latter corresponds to the limit $z\to\infty$ at a fixed P). Let us assume an initial characteristic perturbation of the ion density $\sim\delta n_{\mathrm{i}}>0$ in a region with a characteristic spatial scale $\sim L$. Then an electrostatic potential $\delta\varphi\sim T_{\mathrm{e}}\delta n_{\mathrm{i}}/(n_{\mathrm{i}}e)$ appears in this region since Boltzmann-distributed electrons follow the plasma quasineutrality. This potential corresponds to an electric field $|\delta\mathbf{E}|\sim(T_{\mathrm{e}}\delta n_{\mathrm{i}})/(n_{\mathrm{i}}eL)$. The field is directed outward from the region and induces a mobility-limited ion flow with the velocity $|\delta\mathbf{v}|\sim(T_{\mathrm{e}}\delta n_{\mathrm{i}})/(m_{\mathrm{i}}n_{\mathrm{i}}\nu L)$, which leads to the outgoing ion flux $\sim(LT_{\mathrm{e}}\delta n_{\mathrm{i}})/(m_{\mathrm{i}}\nu)$. Comparing this ion flux with the growth of the number of ions per unit time due to the instability $\sim L^3\alpha_{\mathrm{a}}\delta n_{\mathrm{i}}$, we find the instability length $L\sim\sqrt{T_{\mathrm{e}}/(m_{\mathrm{i}}\nu\alpha_{\mathrm{a}})}$. The threshold wave number is then $k\sim 1/L\sim\sqrt{(m_{\mathrm{i}}\nu\alpha_{\mathrm{a}})/T_{\mathrm{e}}}$ which coincides with Eq. (18) under the assumptions made in this derivation.

C. Collective attraction

The collective attraction derived for an ionization source proportional to the electron density [5–7] has the scaling $\varphi(r)\propto\cos(r/L)/r$, where $\varphi(r)$ is the attractive component of the potential as the function of distance from the grain, and the attraction length L is *exactly* equal to the instability length derived here. Compare Eq. (18) with, e.g., Eq. (27) in Ref. [6]. This can be easily explained. Both the instability length derived here and the attraction length derived in Refs. [5–7] are determined as the reciprocal real wave number at which the static dielectric function crosses zero [5,6]. Hence, the instability mechanism and the mechanism of the collective attraction are essentially the same, and the cosine-type collective attraction of Refs. [5–7] cannot be realized. The same applies to the collective drag force derived in Ref. [16].

For a constant ionization source, in contrast, the collective attraction falls off exponentially with the distance [8,9] and is physically sound since we found no instability for $\gamma=0$.

D. Constant ionization source

Examples of a constant ionization source are metastable pooling [17], photoionization, and grain radioactivity. Also, a constant ionization source can virtually take place in the case of a two-temperature electron distribution [18–20] if the hot component has a sufficiently large temperature so that it does not respond to ion plasma oscillations.

E. Dust-acoustic plasma waves

There have been investigations of dust-acoustic waves in a homogeneous ionization-absorption-balanced complex plasma [21–23]. These investigations assume a quasistatic ion response to perturbations of the dust density. Hence, similar to the collective attraction, the validity of these investigations depends on the stability of the ionization-absorption balance at ion time scales. In Refs. [21–23], the ionization source was assumed to be proportional to the electron density, and this introduces implicit restrictions on the system size. The system size should be less than the instability length derived in the present paper. Hence, the results of these investigations are only valid for wave numbers exceeding substantially the instability threshold wave number derived here.

F. Ion absorption

The parameters η and β depend on the model of ion absorption on dust grains. For reference, we provide expressions for the OML case [1,2]. If the ion current on a single dust grain is given by the OML expression and $\tau \ll z$, we have

$$\eta \approx P\frac{a}{\lambda_{Di}}\frac{1}{\sqrt{2\pi}}, \quad \beta \approx 1. \tag{21}$$

The applicability of the OML expression is discussed, e.g., in Refs. [2,14].

Our model assumes that the ion absorption rate $\alpha_a(Z_d)$ varies only with the dust charge and thus does not depend explicitly on the ion density. This is true, e.g., for the OML case, but—in general—the ion absorption rate may depend explicitly on the ion density. This is shown in Refs. [14,24] where the trapped ion effect on the static ion current on a dust grain is studied. To include this effect in our model, one needs to extend the approach of Refs. [14,24] to the dynamic case.

V. CONCLUSION

We have found that the ionization-absorption balance in a homogeneous and isotropic complex plasma is unstable at ion time scales if the ionization source is proportional to the electron density. The instability length has been found to be exactly equal to the length of the cosine-type collective attraction derived in Refs. [5–7] for an ionization source proportional to the electron density, and hence such collective attraction cannot be realized in real physical systems. At the same time, we have found no instability for constant ionization source, and hence the exponential-type collective attraction derived for the constant ionization source in Refs. [8,9] is physically sound.

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